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K.H. SCHMIDT, GSI
D. VERMEULEN, INST. F. KERNPHYSIK, TH DARMSTADT

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MUTUAL SUPPORT OF MAGICITIES

K.-H. Schmidt

Gesellschaft für Schwerionenforschung, GSI,
Darmstadt, Federal Republic of Germany

D. Vermeulen

Institut für Kernphysik, Techn. Hochschule,
Darmstadt, Federal Republic of Germany

ABSTRACT

A strong correlation of neutron- and proton shell strengths is demonstrated by a systematic comparison of nuclear binding energies. Both, neutron- and proton shells show a maximum strength at doubly magic nuclei and generally fall down drastically with increasing number of additional particles or holes. This mutual support of shell strengths is not reproduced by usually applied theoretical shell corrections. Therefore, the results of shell model calculations, especially in the vicinity of doubly magic nuclei, concerning ground state shell corrections, fission barriers and decay energies, are expected to show appreciable shortcomings. If the variation of shell strengths is taken into account, the interpretation of the separation energies in terms of single particle energies is modified. This modification could solve the 'lead anomaly', the difficulty in describing the ground state shell corrections and the separation energies around ^{208}Pb simultaneously.

INTRODUCTION

Neutron-proton interactions have been discussed for a long time (see e.g. refs. [1,2,3]), mainly in connection with spectroscopic data. However, the role of neutron-proton interactions in the description of ground state masses is not yet as clear. Usually applied theoretical shell corrections do not include this effect.

though in several cases it has been pointed out that the single particle gap of a closed proton shell may be changed if the number of neutrons varies and vice versa (see e.g. refs. [4,5]). Only in a few cases, e.g. in the semiempirical mass table of Liran and Zeldes[6] and in a shell correction approach of Hilf and v. Groote[7], the variation of shell strengths with occupation numbers is included.

In the last years, the knowledge about the binding energies of nuclei has enlarged considerably. Closed shells can now be followed over a long range of nuclei. E.g. for proton rich N = 126 isotones far from the valley of beta stability, remarkable discrepancies have been observed between empirical and theoretical alpha decay energies[8], which were explained by shortcomings of usually applied shell corrections. In the present work, a survey of the strengths of several closed shells all over the nuclide chart is given.

Recently, an explanation for the lead anomaly, the discrepancy in describing the separation energies and the ground state shell corrections in the vicinity of ^{208}Pb consistently, was given by Werner et al.[9] in terms of a quasi particle interaction. The variation of shell strengths, discussed in the present work, enables us to look at the lead anomaly from a quite different point of view.

EXPERIMENTAL EVIDENCE FOR THE MUTUAL SUPPORT OF MAGICITIES

As a closed shell is characterized by a kink in the binding-energy surface as a function of the neutron and the proton number, respectively (see fig. 1), the shell strength can be deduced from the sharpness of this kink in the binding-energy surface which is given by double mass differences. In the shell model, the single particle neutron shell gap G_n for a nucleus with Z protons and N neutrons is given be the double mass difference

$$\begin{aligned} -G_n(Z,N) &= \Delta S_n(Z,N) \\ &= S_n(Z,N+1) - S_n(Z,N) \qquad \text{(a)} \\ &= M(Z,N+1) - 2\,M(Z,N) + M(Z,N-1) \end{aligned}$$

or, in order to avoid the even-odd structure, (see fig. 5):

$$\begin{aligned} -G_n(Z,N) &= 1/2\ \Delta S_{2n}(Z,N) \\ &= 1/2\ (\ S_{2n}(Z,N+2) - S_{2n}(Z,N)\ ) \qquad \text{(b)} \\ &= 1/2\ (\ M(Z,N+2) - 2\,M(Z,N) + M(Z,N-2)\ ), \end{aligned}$$

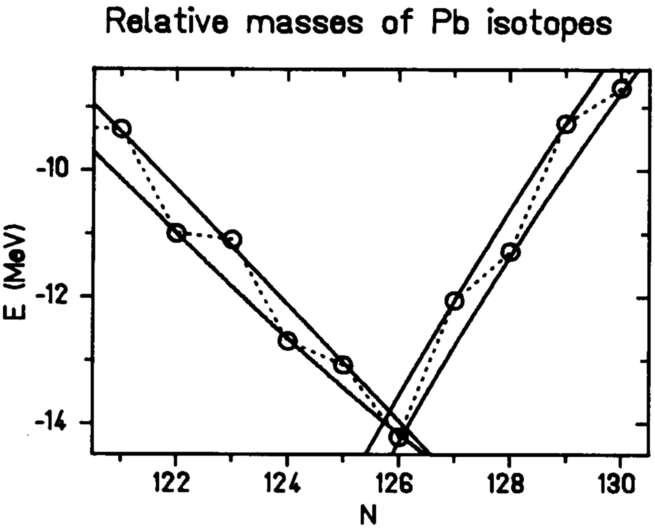

Fig. 1: Experimental masses of Pb-isotopes relative to the values of the droplet model without shell corrections[10]. The full lines are fitted polynomials to the masses of even and odd isotopes below and above N = 126, respectively.

if rearrangement and pairing corrections are neglected. This is demonstrated in fig. 2. A similar relation holds for the single particle proton shell gap G_p. A survey of this quantity in the vicinity of magic nuclei is given in figs. 3 and 4. The empirical masses are taken from ref.[11]. In order to restrict on the single particle properties, the reduced values $\Delta S_{2n}^{red}/2$ are shown which

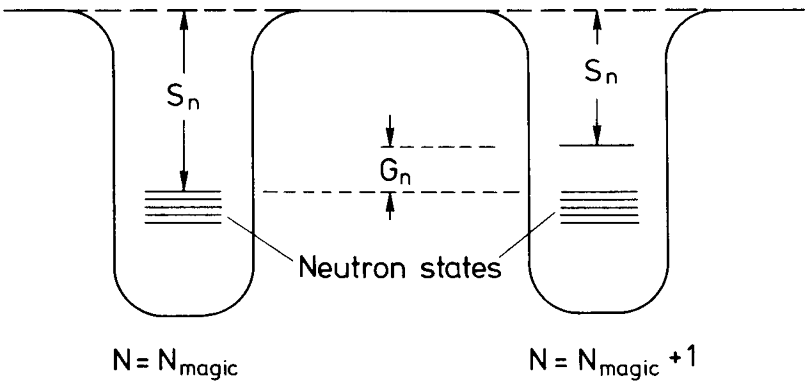

Fig. 2: Schematic diagram, demonstrating that in a shell model descriptions of nuclei without pairing corrections and rearrangement corrections the neutron shell gap G_n is given approximately by the difference of neutron separation energies S_n:

$$-G_n = S_n(Z,N_{magic}+1) - S_n(Z,N_{magic})$$

are corrected for the curvature ΔS_{2n}^{DM} of the binding energy surface as predicted by the droplet model (DM) (ref. [10]) including the Wigner term but without shell corrections:

$$\Delta S_{2n}^{red} = \Delta S_{2n} - \Delta S_{2n}^{DM}$$
$$= (M(Z,N+2) - 2\, M(Z,N) + (M(Z,N-2)) \qquad (c)$$
$$- (M_{DM}(Z,N+2) - 2\, M_{DM}(Z,N) + M_{DM}(Z,N-2)).$$

ΔS_{2p}^{red} is defined in a similar way.

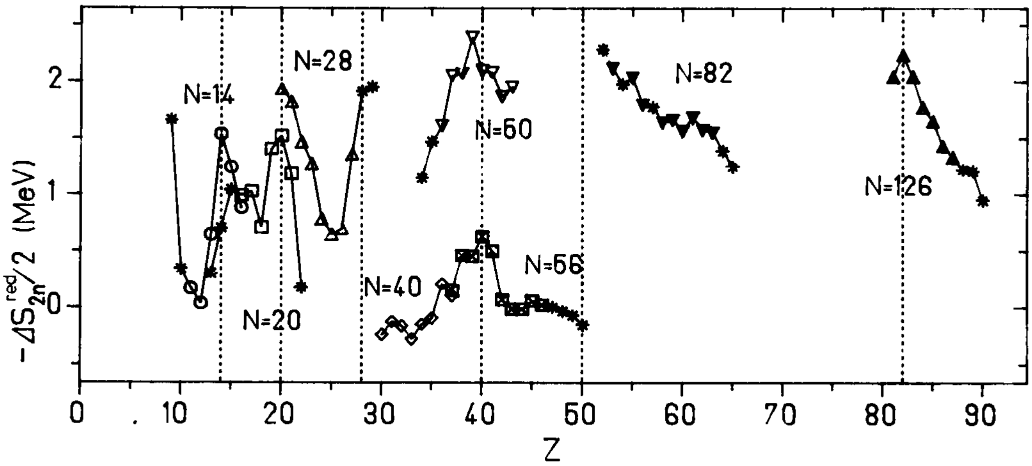

Fig. 3: Half of the difference ΔS_{2n}^{red} between two-neutron separation energies, treated as a function of the proton number Z. The contribution of the droplet mass surface[10] including the Wigner term has been subtracted. The neutron shells are indicated by different symbols. Stars correspond to the cases when the mass (ref. [11]) is taken from systematics and not from experiment. The experimental masses are taken from refs. [11,12,8].

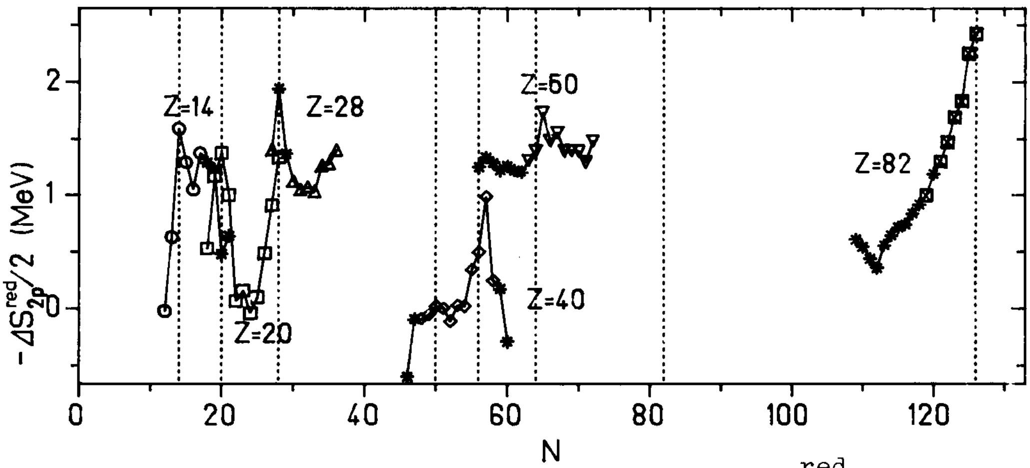

Fig. 4: Same as fig. 3 for half the difference ΔS_{2p}^{red} between two-proton separation energies, treated as a function of the neutron number N.

Figs. 3 and 4 demonstrate that the strengths of nearly all shells change drastically over the nuclide chart. The 82 proton shell, e.g., nearly has disappeared completely in this representation for the most neutron deficient known lead isotopes. Maxima of the double mass differences occur near the proton numbers 14, 20, 28, 40, 50, and 82 as well as near the neutron numbers 14, 20, 28, 40, 50, 56, 64, and 126. Most of them coincide with the well known magic numbers. In all cases where empirical binding energies are available, fig. 3 and 4 suggest that the single particle shell gap of one kind of nucleons tends to grow if the other kind of nucleons approaches a shell closure. Thus, the conclusion may be drawn that the magicities of neutrons and protons generally support each other.

PAIRING CORRELATIONS

The characteristics of pairing correlations are of special interest in this context because they are generally involved in mass systematics and may mask other trends to be investigated. Pairing is especially important in considering shell strengths because the pairing gap tends to counteract the shell correction.

As an example, the values ΔS_n and $\Delta S_{2n}/2$ are compared for isotones with the magic number N = 126 and for isotones with the non magic number N = 130 in fig. 5. For non magic nuclei, the experimental $\Delta S_{2n}/2$ values nearly coincide with the pure droplet

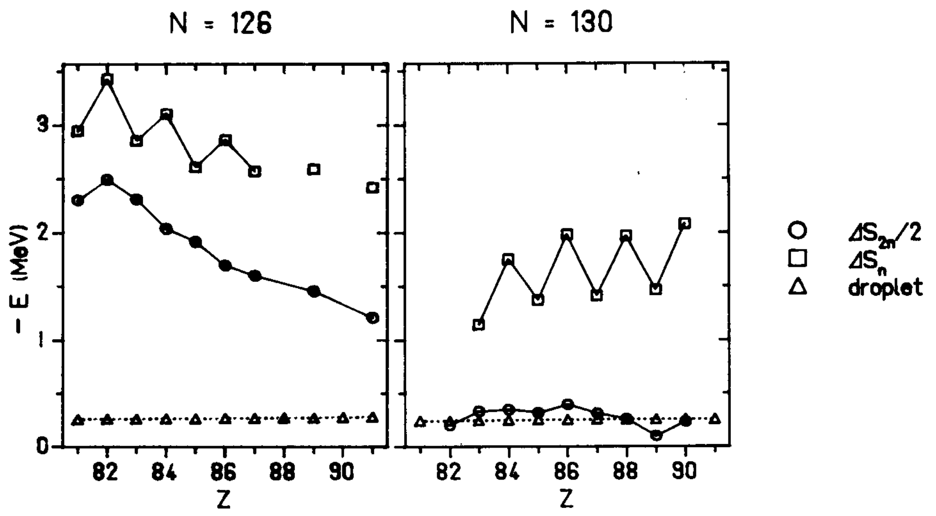

Fig. 5: Double mass differences ΔS_n and $\Delta S_{2n}/2$ for the N = 126 and N = 130 isotones, demonstrating the influence of shell structure and of the pairing interaction on these quantities.

estimation of this quantity. The ΔS_n-values are expected to deviate from the droplet contribution by about twice the neutron pairing gap. Additionally, the ΔS_n-values show an odd-even structure which is usually explained by an attractive interaction between the unpaired proton and the unpaired neutron in odd-odd nuclei (see e.g. discussion of fig. 2.5 in ref. [12]).

Also for N = 126 isotones there is a difference between the values of ΔS_n and $\Delta S_{2n}/2$. According to BCS-calculations of Mosel[14], the proton pairing correction of Pb-isotopes and the neutron pairing correction of 126-neutron isotones in the vicinity of ^{208}Pb are predicted to vanish. Consequently, rather the values of ΔS_n should represent the single particle shell gap than the values of ΔS_{2n}. The empirical masses shown in fig. 1, however, indicate that the neutron pairing correlation of ^{208}Pb does not disappear completely and that in this case the pairing corrected kink in the binding energy surface at N = 126 is represented by a value which lies between the ΔS_n and the $\Delta S_{2n}/2$ values.

DISCUSSION

Modified Interpretation of Separation Energies

So far we identified the shell gap with the difference of separation energies (relations a and b). This is justified, if the neutron shell correction δU_n does not depend on the proton number and vice versa. Then we have the simple relations:

$$\begin{aligned} \delta U_n(Z,N+1) - 2\,\delta U_n(Z,N) + \delta U_n(Z,N-1) &= -\,G_n^{red} \\ \delta U_p(Z,N+1) - 2\,\delta U_p(Z,N) + \delta U_p(Z,N-1) &= 0 \end{aligned} \qquad \text{(d)}$$

The mutual support of magicities, as demonstrated by figs. 3 and 4, leads to an additional reduction of the binding energies of the neighbours of doubly magic nuclei. Consequently, relations (d) no longer hold, and the absolute values of ΔS_n and ΔS_p are enlarged by a correction term C:

$$\begin{aligned} \Delta S_n &= \Delta S_n^{DM} + \delta U_n(Z,N+1) - 2\,\delta U_n(Z,N) + \delta U_n(Z,N-1) \\ &\qquad + \delta U_p(Z,N+1) - 2\,\delta U_p(Z,N) + \delta U_p(Z,N-1) \\ &= \Delta S_n^{DM} - G_n^{red} - C_n \end{aligned} \qquad \text{(e)}$$

The magnitude of C will be discussed later, but generally we expect

$$G_n < |\Delta S_n| \quad \text{and} \quad G_p < |\Delta S_p|.$$

The discrepancy is expected to grow with the amount of the variation of the shell strength.

This correction must also be applied to the data in figs. 3 and 4 if shell gaps are to be deduced, but it is obvious that it may only change the magnitude of the slopes but not the sign. Thus, the general statement of a mutual support of magicities still holds.

Connections to the Lead Anomaly

In fig. 6, the ground state shell corrections δU of some nuclei around $N = 126$ are shown. The experimental values are compared with theoretical shell corrections, which were obtained from Nilsson model calculations with Strutinski renormalisation and BCS-pairing correlations (ref. [15]). In order to illustrate qualitatively the influence of the variation of shell strengths, we show the result of a simple calculation. In this calculation, the shell correction was obtained by a simplified Strutinski procedure, using a uniformly distributed level sequence which was bunched at a shell closure. For simplicity, the shell gaps around ^{208}Pb were assumed to follow an exponential law:

$$G_n^{red} = G_p^{red} = 2.1\ \text{MeV} * \exp(-(|Z-82| + |N-126|)/20) \qquad \text{(f)}$$

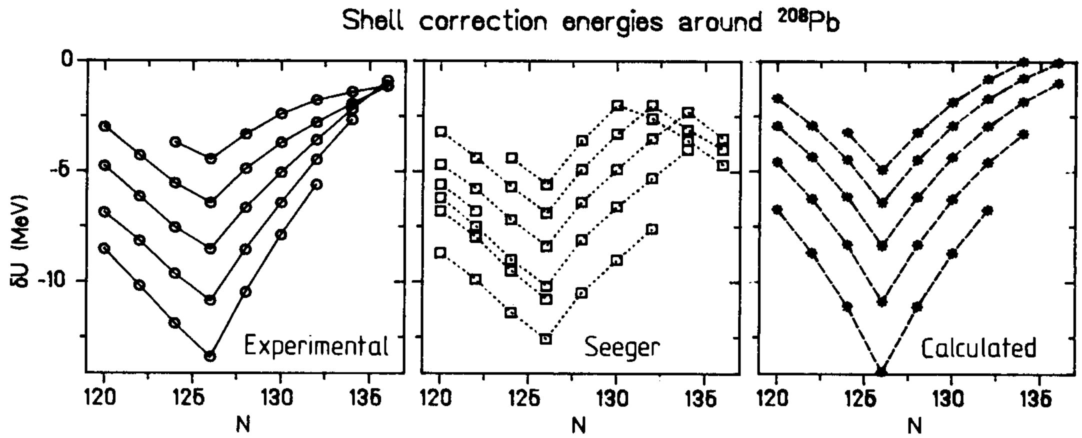

Fig. 6: Comparison of shell correction energies around N = 126. The lines connect the values of Pb, Po, Rn, Ra, and Th-isotopes. $\delta U_{experimental}$ is determined as the difference of the empirical mass[11] and the droplet mass[10] without shell corrections. δU_{Seeger} is the result of a Nilsson-model calculation with Strutinski renormalisation and BCS pairing-correlations[15]. For $\delta U_{calculated}$ the mutual support of magicities was taken into account (see text).

With these parameters, the experimental shell corrections as well as the separation energies in the vicinity of ^{208}Pb can approximately be reproduced. The main feature of these calculated shell corrections is the flattening of the valley at N = 126 with increasing distance from ^{208}Pb, which is not reproduced by Seeger's calculation. This flattening already sets in for the spherical nuclei, directly neighboured to ^{208}Pb (compare figs. 3 and 4) and cannot be explained by the onset of a static deformation.

Usually, the single particle shell gaps are determined by relation a). In ref. [13] the following values are given for ^{208}Pb:

$$G_n = 3.44 \text{ MeV} \quad \text{and} \quad G_p = 4.23 \text{ MeV}.$$

In relation (f), however, appreciable smaller values for the shell gaps were introduced:

$$G_n = G_n^{red} + G_n^{DM} = 2.1 \text{ MeV} + 0.26 \text{ MeV} = 2.36 \text{ MeV} \quad \text{and}$$

$$G_p = G_p^{red} + G_p^{DM} = 2.1 \text{ MeV} + 0.88 \text{ MeV} = 2.98 \text{ MeV}.$$

Consequently, C_n = 1.08 MeV and C_p = 1.25 MeV. Brack et al.[16] introduced an artificial reduction of the shell gaps for ^{208}Pb of about the same amount (1.5 MeV) in order to reproduce the experimental shell effects.

We think that we can explain this reduction by a modified interpretation of the relation between separation energies and single particle levels. The consideration of the mutual support of magicities requires a correction term which reduces the ground state shell correction of ^{208}Pb by about 1/3, compared to a shell model calculation which is adapted to the separation energies[16].

Consequences for Mass Predictions

The shell correction approaches of Myers[10] and v. Groote et al.[17] as well as Nilsson model calculations with Strutinski renormalisation[15] yield a nearly constant value for the strength of each shell. The same holds for the result of a spherical Hartree-Fock calculation[18]. Therefore, the predictions of these models for several quantities are expected to show some shortcomings. The extension of the peak of additional nuclear binding energy due to the ground state shell correction around doubly magic nuclei is smaller than calculated. By this effect the calculated fission barriers are influenced directly[19]. Especially the suspected island of superheavy nuclei may be considerably smaller than expected. In addition, calculated decay energies, e.g. the Q_α and Q_β values of superheavy nuclei, are also affected.

CONCLUSION

It has been shown, that the correlation between proton and neutron magicities shows clearly up in nuclear ground state masses. The shell correction approaches usually applied do not account for this effect. It has been pointed out that the problems in describing the ground state shell correction of ^{208}Pb and the separation energies around ^{208}Pb consistently may be solved, if the mutual support of neutron and proton magicities is taken into account.

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